Experiment 13 – Sampling and reconstruction

Preliminary discussion
So far, the experiments in this manual have concentrated on communications systems that transmit analog signals. However, digital transmission is fast replacing analog in commercial communications applications. There are several reasons for this including the ability of digital signals and systems to resist interference caused by electrical noise.

Many digital transmission systems have been devised and several are considered in later experiments. Whichever one is used, where the information to be transmitted (called the message) is an analog signal (like speech and music), it must be converted to digital first. This involves sampling which requires that the analog signal’s voltage be measured at regular intervals.

Figure 1a below shows a pure sinewave for the message. Beneath the message is the digital sampling signal used to tell the sampling circuit when to measure the message. Beneath that is the result of “naturally” sampling the message at the rate set by the sampling signal. This type of sampling is “natural” because, during the time that the analog signal is measured, any change in its voltage is measured too. For some digital systems, a changing sample is unacceptable. Figure 1b shows an alternative system where the sample’s size is fixed at the instant that the signal is measured. This is known as a sample-and-hold scheme (and is also referred to as pulse amplitude modulation).

![Figure 1a](image1a.png) ![Figure 1b](image1b.png)
Regardless of the sampling method used, by definition it captures only pieces of the message. So, how can the sampled signal be used to recover the whole message? This question can be answered by considering the mathematical model that defines the sampled signal:

\[ \text{Sampled message} = \text{the sampling signal} \times \text{the message} \]

As you can see, sampling is actually the multiplication of the message with the sampling signal. And, as the sampling signal is a digital signal which is actually made up of a DC voltage and many sinewaves (the fundamental and its harmonics) the equation can be rewritten as:

\[ \text{Sampled message} = (\text{DC} + \text{fundamental} + \text{harmonics}) \times \text{message} \]

When the message is a simple sinewave (like in Figure 1) the equation’s solution (which necessarily involves some trigonometry that is not shown here) tells us that the sampled signal consists of:

- A sinewave at the same frequency as the message
- A pair of sinewaves that are the sum and difference of the fundamental and message frequencies
- Many other pairs of sinewaves that are the sum and difference of the sampling signals’ harmonics and the message

This ends up being a lot of sinewaves but one of them has the same frequency as the message. So, to recover the message, all that need be done is to pass the sampled signal through a low-pass filter. As its name implies, this type of filter lets lower frequency signals through but rejects higher frequency signals.

That said, for this to work correctly, there's a small catch which is discussed in Part E of the experiment.

The experiment
In this experiment you'll use the Emona DATEx to sample a message using natural sampling then a sample-and-hold scheme. You'll then examine the sampled message in the frequency domain using the NI ELVIS Dynamic Signal Analyzer. Finally, you'll reconstruct the message from the sampled signal and examine the effect of a problem called aliasing.

It should take you about 50 minutes to complete this experiment.
Equipment

- Personal computer with appropriate software installed
- NI ELVIS plus connecting leads
- NI Data Acquisition unit such as the USB-6251 (or a 20MHz dual channel oscilloscope)
- Emona DATCH experimental add-in module
- two BNC to 2mm banana-plug leads
- assorted 2mm banana-plug patch leads

Part A – Sampling a simple message
The Emona DATCH has a Dual Analog Switch module that has been designed for sampling. This part of the experiment lets you use the module to sample a simple message using two techniques.

Procedure

1. Ensure that the NI ELVIS power switch at the back of the unit is off.
2. Carefully plug the Emona DATCH experimental add-in module into the NI ELVIS.
3. Set the Control Mode switch on the DATCH module (top right corner) to PC Control.
4. Check that the NI Data Acquisition unit is turned off.
5. Connect the NI ELVIS to the NI Data Acquisition unit (DAQ) and connect that to the personal computer (PC).
6. Turn on the NI ELVIS power switch at the back then turn on its Prototyping Board Power switch at the front.
7. Turn on the PC and let it boot-up.
8. Once the boot process is complete, turn on the DAQ then look or listen for the indication that the PC recognise it.
9. Launch the NI ELVIS software.
10. Launch the DATCH soft front-panel (SFP).
11. Check you now have soft control over the DATCH by activating the PCM Encoder module's soft PDM/TDM control on the DATCH SFP.
Note: If your set-up is working correctly, the PCM Decoder module’s LED on the DATEx board should turn on and off.

12. Connect the set-up shown in Figure 2 below.

Note: Insert the black plugs of the oscilloscope leads into a ground (GND) socket.

![Figure 2](image)

This set-up can be represented by the block diagram in Figure 3 below. It uses an electronically controlled switch to connect the message signal (the 2kHz SINE output from the Master Signals module) to the output. The switch is opened and closed by the 8kHz DIGITAL output of the Master Signals module.

![Figure 3](image)
13. Launch the NI ELVIS Oscilloscope VI.

14. Set up the scope per the procedure in Experiment 1 (page 1-13) ensuring that the Trigger Source control is set to CH A.

15. Adjust the scope’s Timebase control to view two or so cycles of the Master Signals module’s 2kHz SINE output.

16. Activate the scope’s Channel B input by pressing the Channel B Display control’s ON/OFF button to observe the sampled message out of the Dual Analog Switch module as well as the message.

   Tip: To see the two waveforms clearly, you may need to adjust the scope so that the two signals are not overlayed.

17. Draw the two waveforms to scale in the space provided on the next page leaving room to draw a third waveform.

   Tip: Draw the message signal in the upper third of the graph and the sampled signal in the middle third.

**Question 1**
What type of sampling is this an example of?

☐ Natural

☐ Sample-and-hold

**Question 2**
What two features of the sampled signal confirm this?
Ask the instructor to check your work before continuing.
18. Modify the set-up as shown in Figure 4 below.

Before you do...
The set-up in Figure 4 below builds on the set-up that you’ve already wired so don’t pull it apart. To highlight the changes that we want you to make, we’ve shown your existing wiring as dotted lines.

This set-up can be represented by the block diagram in Figure 5 on the next page. The electronically controlled switch in the original set-up has been substituted for a sample-and-hold circuit. However, the message and sampling signals remain the same (that is, a 2kHz sinewave and an 8kHz pulse train).
19. Draw the new sampled message to scale in the space that you left on the graph paper.

**Question 3**
What two features of the sampled signal confirm that the set-up models the sample-and-hold scheme?

__________________________________________________________________________

__________________________________________________________________________

Ask the instructor to check your work before continuing.
Part B - Sampling speech
This experiment has sampled a 2kHz sine wave. However, the message in commercial digital communications systems is much more likely to be speech and music. The next part of the experiment lets you see what a sampled speech signal looks like.

20. Disconnect the plugs to the Master Signals module’s 2kHz SINE output.

21. Connect them to the Speech module’s output as shown in Figure 6 below.

   **Remember:** Dotted lines show leads already in place.

22. Set the scope’s Timebase control to the 500μs/div position.

23. Hum and talk into the microphone while watching the scope’s display.

   ![Figure 6](image)

   **Figure 6**

   Ask the instructor to check your work before continuing.
Part C - Observations and measurements of the sampled message in the frequency domain
Recall that the sampled message is made up of many sinewaves. Importantly, for every
sinewave in the original message, there’s a sinewave in the sampled message at the same
frequency. This can be proven using the NI ELVIS Dynamic Signal Analyzer. This device
performs a mathematical analysis called Fast Fourier Transform (FFT) that allows the
individual sinewaves that make up a complex waveform to be shown separately on a frequency-
domain graph. The next part of the experiment lets you observe the sampled message in the
frequency domain.

24. Return the scope’s Timebase control to the 100µs/div position.

25. Disconnect the plugs to the Speech module’s output and reconnect them to the Master
Signals module’s 2kHz SINE output.

   **Note:** The scope should now display the waveform that you drew for Step 19.

26. Suspend the scope VI’s operation by pressing its RUN control once.

   **Note:** The scope’s display should freeze.

27. Launch the NI ELVIS Dynamic Signal Analyzer VI.

   **Note:** If the Dynamic Signal Analyzer VI has launched successfully, your display should
look like Figure 7 below.

![Figure 7](image_url)
28. Adjust the Signal Analyzer’s controls as follows:

**General**
Sampling to Run

**Input Settings**
- **Source Channel** to **Scope CHB**
- **Voltage Range** to ±10V

**FFT Settings**
- **Frequency Span** to 40,000
- **Resolution** to 400
- **Window** to 7 Term B-Harris

**Averaging**
- **Mode** to RMS
- **Weighting** to Exponential
- **# of Averages** to 3

**Triggering**
- **Triggering** to **Source Channel**

**Frequency Display**
- **Units** to dB (for now)
- **Markers** to OFF (for now)
- **RMS/Peak** to RMS
- **Scale** to Auto

**Note:** If the Signal Analyzer VI has been set up correctly, your display should look like Figure 8 below.
If you've not attempted Experiment 7, the Signal Analyzer’s display may need a little explaining here. There are actually two displays, a large one on top and a much smaller one underneath. The smaller one is a time domain representation of the input (in other words, the display is a scope).

The larger of the two displays is the frequency domain representation of the complex waveform on its input (the sampled message). The humps represent the sinewaves and, as you can see, the sampled message consists of many of them. As an aside, these humps should just be simple straight lines, however, the practical implementation of FFT is not as precise as the theoretical expectation.

If you have done Experiment 7, go directly to Step 36 on the next page.

29. Activate the Signal Analyzer’s markers by pressing the Markers button.

   **Note 1:** When you do, the button should display the word “ON” instead of “OFF”.

   **Note 2:** Green horizontal and vertical lines should appear on the Signal Analyzer’s frequency domain display. If you can’t see both lines, turn the Markers button off and back on a couple of times while watching the display.

The NI ELVIS Dynamic Signal Analyzer has two markers M1 and M2 that default to the left side of the display when the NI ELVIS is first turned on. They’re repositioned by “grabbing” their vertical lines with the mouse and moving the mouse left or right.

30. Use the mouse to grab and slowly move marker M1.

   **Note:** As you do, notice that marker M1 moves along the Signal Analyzer’s trace and that the vertical and horizontal lines move so that they always intersect at M1.

31. Repeat Step 30 for marker M2.

The NI ELVIS Dynamic Signal Analyzer includes a tool to measure the difference in magnitude and frequency between the two markers. This information is displayed in green between the upper and lower parts of the display.

32. Move the markers while watching the measurement readout to observe the effect.

33. Position the markers so that they’re on top of each other and note the measurement.

   **Note:** When you do, the measurement of difference in magnitude and frequency should both be zero.
Usefully, when one of the markers is moved to the extreme left of the display, its position on the X-axis is zero. This means that the marker is sitting on 0Hz. It also means that the measurement readout gives an absolute value of frequency for the other marker. This makes sense when you think about it because the readout gives the difference in frequency between the two markers but one of them is zero.

34. Move \textit{M2} to the extreme left of the display.

35. Align \textit{M1} with the highest point of any one of the humps.

\textbf{Note:} The readout will now be showing you the frequency of the sinewave that the hump represents.

Recall that the message signal being sampled is a 2kHz sinewave. This means that there should also be a 2kHz sinewave in the sampled message.

36. Use the Signal Analyzer's \textit{M1} marker to locate sinewave in the sampled message that has the same frequency as the original message.

As discussed earlier, the frequency of all of the sinewaves in the sampled message can be mathematically predicted. Recall that digital signals like the sampling circuit's clock signal are made up out of a DC voltage and many sinewaves (the fundamental and harmonics). As this is a sample-and-hold sampling scheme, the digital signal functions as a series of pulses rather than a squarewave. This means that the sampled signal's spectral composition consists of a DC voltage, a fundamental and both even and odd whole number multiples of the fundamental. For example, the 8kHz sampling rate of your set-up consists of a DC voltage, an 8kHz sinewave (fs), a 16kHz sinewave (2fs), a 24kHz sinewave (3fs) and so on.

The multiplication of the sampling signal's DC component with the sinewave message gives a sinewave at the same frequency as the message and you have just located this in the sampled signal's spectrum.

![Checkmark] Ask the instructor to check your work before continuing.
The multiplication of the sampling signal’s fundamental with the sinewave message gives a pair of sinewaves equal to the fundamental frequency plus and minus the message frequency. That is, it gives a 6kHz sinewave (8kHz – 2kHz) and a 10kHz sinewave (8kHz + 2kHz).

In addition to this, the multiplication of the sampling signal’s harmonics with the sinewave message gives pairs of sinewaves equal to the harmonics’ frequency plus and minus the message frequency. That is, the signal also consists of sinewaves at the following frequencies: 14kHz (16kHz – 2kHz), 18kHz (16kHz + 2kHz), 22kHz (24kHz – 2kHz), 26kHz (24kHz + 2kHz) and so on.

All of these sum and difference sinewaves in the sampled signal are appropriately known as aliases.

37. Use the Signal Analyzer’s M1 marker to locate and measure the exact frequency of the sampled signal’s first six aliases. Record your measurements in Table 1 below.

Tip: Their frequencies will be close to those listed above.

<table>
<thead>
<tr>
<th>Alias 1</th>
<th>Alias 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alias 2</td>
<td>Alias 5</td>
</tr>
<tr>
<td>Alias 3</td>
<td>Alias 6</td>
</tr>
</tbody>
</table>

Ask the instructor to check your work before continuing.

Why aren’t the alias frequencies exactly as predicted?
You will have notice that the measured frequencies of your aliases don’t exactly match the theoretically predicted values. This is not a flaw in the theory. To explain, the Emona DATEX has been designed so that the signals out of the Master Signals module are synchronised. This is a necessary condition for the implementation of many of the modulation schemes in this manual. To achieve this synchronisation, the 8kHz and 2kHz signals are derived from a 100kHz master crystal oscillator. As a consequence, their frequencies are actually 8.3kHz and 2.08kHz.
Part D – Reconstructing a sampled message
Now that you have proven that the sampled message consists of a sinewave at the original message frequency, it’s easy to understand how a low-pass filter can be used to “reconstruct” the original message. The LPF can pick-out the sinewave at the original message frequency and reject the other higher frequency sinewaves. The next part of the experiment lets you do this.

38. Suspend the Signal Analyzer VI’s operation by pressing its RUN control once.

   **Note:** The scope’s display should freeze.

39. Restart the scope’s VI by pressing its RUN control once.

40. Locate the Tuneable Low-pass Filter module on the DATEx SFP and set its soft Gain control to about the middle of its travel.

41. Turn the Tuneable Low-pass Filter module’s soft Cut-off Frequency Adjust control fully anti-clockwise.

42. Modify the set-up as shown in Figure 9 below.

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**Figure 9**
The set-up in Figure 9 can be represented by the block diagram in Figure 10 below. The Tuneable Low-pass Filter module is used to recover the message. The filter is said to be "tuneable" because the point at which frequencies are rejected (called the cut-off frequency) is adjustable.

At this point there should be nothing out of the Tuneable Low-pass Filter module. This is because it has been set to reject almost all frequencies, even the message. However, the cut-off frequency can be increased by turning the module's Cut-off Frequency Adjust control clockwise.

43. Slowly turn the Tuneable Low-pass Filter module’s soft Cut-off Frequency control clockwise and stop when the message signal has been reconstructed and is roughly in phase with the original message.

Ask the instructor to check your work before continuing.
Part E - Aliasing
At present, the filter is only letting the message signal through to the output. It is comfortably rejecting all of the other sinewaves that make up the sampled message (the aliases). This is only possible because the frequency of these other sinewaves is high enough. Recall from your earlier measurements that the lowest frequency alias is 6kHz.

Recall also that the frequency of the aliases is set by the sampling signal’s frequency (for a given message). So, suppose the frequency of the sampling signal is lowered. A copy of the message would still be produced because that’s a function of the sampling signal’s DC component. However, the frequency of the aliases would all go down. Importantly, if the sampling signal’s frequency is low enough, one or more of the aliases pass through the filter along with the message. Obviously, this would distort the reconstructed message which is a problem known as aliasing.

To avoid aliasing, the sampling signal’s theoretical minimum frequency is twice the message frequency (or twice the highest frequency in the message if it contains more than one sinewave and is a baseband signal). This figure is known as the Nyquist Sample Rate and helps to ensure that the frequency of the non-message sinewaves in the sampled signal is higher than the message’s frequency. That said, filters aren’t perfect. Their rejection of frequencies beyond the cut-off is gradual rather than instantaneous. So in practice the sampling signal’s frequency needs to be a little higher than the Nyquist Sample Rate.

The next part of the experiment lets you vary the sampling signal’s frequency to observe aliasing.

44. Slide the NI ELVIS Function Generator’s Control Mode switch so that it’s no-longer in the Manual position.

45. Launch the Function Generator’s VI.

46. Press the Function Generator VI’s ON/OFF control to turn it on.

47. Adjust the Function Generator for an 8kHz output.

Note: It’s not necessary to adjust any other controls as the Function Generator’s SYNC output will be used and this is a digital signal.
48. Modify the set-up as shown in Figure 11 below.

This set-up can be represented by the block diagram in Figure 12 below. Notice that the sampling signal is now provided by the Function Generator which has an adjustable frequency.
At this point, the sampling of the message and its reconstruction should be working as before.

49. Set the scope's Timebase control to the 500µs/div position.

50. Reduce the frequency of the Frequency Generator's output by 1000Hz and observe the effect this has (if any) on the reconstructed message signal.

   Note: Give the Function Generator time to output the new frequency before you change it again.

51. Disconnect the scope's Channel B input from the Tuneable Low-pass Filter module's output and connect it to the Dual Analog Switch module's S&H output.

52. Suspend the scope VI's operation.

53. Restart the Signal Analyzer's VI.

**Question 4**
What has happened to the sampled signal's aliases?

54. Suspend the Signal Analyzer VI's operation.

55. Restart the scope's VI.

56. Return the scope's Channel B input to the Tuneable Low-pass Filter module's output.

57. Repeat Steps 50 to 56 until the Function Generator's output frequency is 3000Hz.

**Question 5**
What's the name of the distortion that appears when the sampling frequency is low enough?

**Question 6**
What happens to the sampled signal's lowest frequency alias when the sampling rate is 4kHz?
58. If you’ve not done so already, repeat Steps 54 to 56.

59. Increase the frequency of the Frequency Generator’s output in 200Hz steps and stop the when the recovered message is a stable, clean copy of the original.

60. Record this frequency in Table 2 below.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum sampling frequency (without aliasing)</td>
<td></td>
</tr>
</tbody>
</table>

Question 7
Given the message is a 2kHz sinewave, what’s the theoretical minimum frequency for the sampling signal? Tip: If you’re not sure, see the notes on page 13-18.

Question 8
Why is the actual minimum sampling frequency to obtain a reconstructed message without aliasing distortion higher than the theoretical minimum that you calculated for Question 5?

Ask the instructor to check your work before finishing.